1.8: Scientific Notation

To write 1 trillion (1 followed by 12 zeros) or 1 googol (1 followed by 100 zeroes) takes a lot of space and time. There is a mathematical scientific notation which is very useful for writing very big and very small numbers.

Example 1.8.1

Large numbers in scientific notation:

a. 1 trillion is written as 1×10^{12} in scientific notation.
b. 4 trillion is written as 4×10^{12} in scientific notation.
c. 1 googol is written as 1×10^{100} in scientific notation.

Example 1.8.2

a. 0.00000547 is written as 5.47×10^{-6} in scientific notation.
b. 0.00031 is written as 3.1×10^{-4} in scientific notation.

The number 45,600,000 is a large number, and, basically is $4.56 \times 10,000,000$ So, it can be written as 4.56×10^7

Similarly, if we consider the number 0.00006772.This is a small number which is $6.772 \times \frac{1}{100000}$. That is, it can be written as 6.772×10^{-5}.

The numbers 4.56×10^7 and 6.772×10^{-5} are said to be written in scientific
notation because the number before the power of 10 is greater than (or equal) to 1 and less than 10, and the decimal number is followed by multiplication by a power of 10.

Standard Form and Scientific Notation

Standard form	Scientific notation
$34,500,000,000$	3.45×10^{10}
0.00000000889	8.89×10^{-9}

Recall from chapter 3 how multiplying or dividing a decimal number by $10, 100, 100, \ldots$ affects the position of the decimal point.

Example 1.8.3

The given numbers are not in scientific notation. Modify each so that your answer is in scientific notation:

a. $1500 = 1.5 \times 10^3$
b. $225000 = 2.25 \times 10^5$
c. $0.0155 = 1.55 \times 10^{-2}$
d. $0.00000094 = 9.4 \times 10^{-7}$

Example 1.8.4

The given numbers are not in scientific notation (look at the decimal number and see that it is either less than 1 or greater than 10). Modify each so that your answer is in scientific notation:

a. $56.7 \times 10^8 = 5.67 \times 10^9$
b. $88.9 \times 10^{-7} = 8.89 \times 10^{-6}$
c. $0.55 \times 10^9 = 5.5 \times 10^8$
d. $0.88 \times 10^{-4} = 8.8 \times 10^{-5}$

Helpful tip: Note that the given numbers were not in scientific notation because the decimal number was either greater than 10 or less than 1. To modify the decimal and rewrite the given number in scientific notation, we either increase its size, and thus we must decrease the size of the exponent, or, we decrease its size, and thus, we must increase the size of the exponent.

Multiplication and Division using Scientific Notation

By grouping the decimal numbers together, and the power of 10 terms together, it becomes easy to multiply and divide numbers in scientific notation. First, we need to recall the properties of exponents (we only need base 10 for this section):

Properties of Exponents (for base 10)

1. Product Property $10^m \cdot 10^n = 10^{m+n}$

Examples: $10^2 \cdot 10^5 = 10^{2+5} = 10^7$ and $10^{-9} \cdot 10^3 = 10^{-9+3} = 10^{-6}$

2. Quotient Property $\frac{10^m}{10^n} = 10^{m-n}$

Examples: $\frac{10^5}{10^3} = 10^{5-3} = 10^2$ and $\frac{10^5}{10^{-4}} = 10^{5-(-4)} = 10^{5+4} = 10^9$

Example 1.8.5

Perform the given operation:

a. $\left(2.3 \times 10^8\right)\left(3 \times 10^{-4}\right) = (2.3 \cdot 3) \times \left(10^8 \cdot 10^{-4}\right) = (2.3 \cdot 3) \times 10^{8+(-4)} = 6.9 \times 10^4$

b. $\frac{6.4 \times 10^{-9}}{3.2 \times 10^{-5}} = \frac{6.4}{3.2} \times \frac{10^{-9}}{10^{-5}} = \frac{6.4}{3.2} \times 10^{-9-(-5)} = 2 \times 10^{-4}$

Helpful tip: Notice how when we multiplied, we added the exponents, and when we divided, we subtracted the exponent in the denominator from the exponent in the numerator. We simply followed the exponent rules.

Example 1.8.6

Perform the given operation and write your answer in scientific notation:

a. $\left(6.2 \times 10^8\right)\left(3.0 \times 10^7\right) = 6.2 \cdot 3 \times 10^{8+7} = 18.6 \times 10^{15} = 1.86 \times 10^{16}$

b. $\frac{\left(4 \times 10^5\right)}{8 \times 10^{-3}} = \frac{4}{8} \times 10^{5-(-3)} = 0.5 \times 10^8 = 5.0 \times 10^7$

Note: Writing 5.0×10^7 is the same as writing 5×10^7. They are interchangeable.

Example 1.8.7

Perform the given operation and write your answer in scientific notation:

$\frac{\left(2.1 \times 10^3\right)\left(3.2 \times 10^{-8}\right)}{\left(2 \times 10^4\right)\left(3 \times 10^9\right)} = \frac{6.72 \times 10^{-5}}{6 \times 10^{13}} = 1.12 \times 10^{-18}$

Example 1.8.8

The debt of a nation is 7 trillion dollars, and there are 300 thousand inhabitants. If the debt was distributed evenly among all the inhabitants, how much would each person have to pay to pay off the debt?

To answer this question we have to divide: $\frac{7 \text{ trillion dollars}}{300 \text{ million people}}$.

Since we are dealing with large numbers, we change each to scientific notation and perform the division to find the amount to be paid per person:

$\frac{7 \times 10^{15}}{3 \times 10^{11}}$ dollars per person $= \frac{7}{3} \times 10^4$ dollars per person $\approx 2.3333 \times 10^4$ dollars per person.

In standard form, the amount is $\$23,333$ per person.

Exit Problem

Compute and write the answer in scientific notation:

$$\frac{\left(6.2 \times 10^2\right)\left(1.5 \times 10^{-4}\right)}{3.1 \times 10^{-9}}$$