1.21: Rewriting Formulas

Michael Faraday (1791 - 1867) was an English scientist who contributed to the fields of electromagnetism and electrochemistry. His mathematical abilities, however, were limited and so he mainly relied on expressing his ideas in clear and simple writing. Later, the scientist James Clerk Maxwell came along and took the work of Faraday (and others), and summarized it in a set of equations known as “Maxwell’s Equations”. His equations are accepted as the basis of all modern electromagnetic theory and take on many different mathematical forms.

The language of mathematics is powerful. It is a language which has the ability to express relationships and principles *precisely* and *succinctly*. Faraday was a brilliant scientist who made history-making discoveries yet they were not truly appreciated until Maxwell was able to translate them into a workable language, that of mathematics.

Definition of a Formula

A formula is a mathematical relationship expressed in symbols.

For example let's consider Einstein's $E=mc^2$ (arguably the most famous formula in the world). This formula is an equation which describes the relationship between the energy a body transmits in the form of radiation (the E) and its mass (the m) along with the speed of light in vacuum (the c). It says that a body of mass m emits energy of the amount E which is precisely equal to mc^2 It can also say that if a body emits an amount of energy E, then its mass m must be E/c^2. We can say this because we solved $E=mc^2$ for m, that is, we rewrote the formula in terms of a specific variable.

Example 19.1

More familiar formulas:

a. $A=lw$ Area of a rectangle $=$ length $\cdot$ width.
b. $C=2\pi r$ Circumference of a circle $=2\cdot\pi\cdot$ radius.
c. $S=d/t$ Speed $=$ distance traveled $\div$ time.

We note that the formula $C=2\pi r$ is "solved" for C since C is by itself on one side of the equation. Suppose we know the circumference of a circle C is equal to 5 inches and we wish to find its radius r. One way to do this is to begin by solving the formula $C=2\pi r$ and then substituting the known values.

Begin with:

$$C=2\pi r$$

and divide both sides by 2π to get

$$\frac{C}{2\pi}=r$$

Next we substitute the value $C=5$ and π to get

$$r=\frac{5}{2\pi}\approx 0.8 \text{ in rounded to the nearest tenth}$$

So $r=0.8$ inches when we round to the nearest tenth.

Solving an Equation for a Specified Variable

1. Use the distributive property of multiplication (if possible).
2. Combine any like terms (if possible).
3. Get rid of denominators (if possible).
4. Collect all terms with the variable you wish to solve for one side of the equation. Do this by using the addition property of equality.
5. Use the distributive property and the multiplication property of equality to isolate the desired variable.

Example 19.2

Solve the following equations for the specific variable indicated in parenthesis.

a) $E = IR($ for $I)$

Divide both sides by R to get $I = \frac{E}{R}$

b) $PV = nRT$ (for R)

Divide both sides by nT to get $R = \frac{PV}{nT}$.

c) $S = \frac{T}{P}($ for $P)$

Multiply each side by P to get $PS = T$ then divide each side by S to get $P = \frac{T}{S}$

d) $V = \frac{MN}{4Z}($ for $N)$

Multiply each side by $4Z$ to get $4ZV = MN$, then divide each side by M to get $N = \frac{4ZV}{M}$

e) $d = a + b + c($ for $b)$

Subtract a and c from both sides to get $b = d - a - c$

f) $B = \frac{zI^2 p}{W}($ for $p)$

Multiply each side by W to get $WB = zI^2 p$, then divide both sides by zI^2 to get $p = \frac{WB}{zI^2}$

g) $P = S - c$ (for c)

Add c to both sides to get $c + P = S$, then subtract P from both sides to get $c = S - P$

h) $A = EC + G$ (for C)

Subtract G from both sides to get $A - G = EC$, then divide both sides by E to get $C = \frac{A-G}{E}$.

i) $w = 2x - 3y$ (for x)

Add $3y$ to both sides to get $w + 3y = 2x$, then divide both sides by 2 to get $\frac{w+3y}{2} = x$, and so, $x = \frac{w+3y}{2}$

Exit Problem

The formula for the area of a triangle is $A = \frac{1}{2}bh.$ Solve this equation for b